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ABSTRACT

This report, part of a series of documents which describe assessment of student learning in various curriculum areas, focuses on assessment of student learning in the mathematics classroom, beginning with an overview of current curriculum goals in mathematics. The rationale for reforming the content, instructional practices, and assessment methods in school mathematics is discussed. Subsequent sections describe alternative methods of assessing mathematics learning that are consistent with the new curriculum and instructional goals set forth for mathematics education. The performance-based assessment methods reviewed are: (1) open-ended questions; (2) mathematical investigations and projects; (3) writing activities in mathematics; (4) observations and interviews; (5) enhanced multiple-choice questions; and (6) portfolio assessments. A description of various methods for scoring and grading student work follows information on assessment methods. A brief summary is provided of activities in performance assessment from selected states. Appendix A contains a focused holistic scoring point scale. (SLD)

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**ASSESSMENT OF STUDENT LEARNING IN  
MATHEMATICS**

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in the Assessment of Student Learning**

**March 1993**

**The South Carolina Center for Excellence in the Assessment of Student Learning was  
established by the South Carolina Commission on Higher Education and  
the College of Education, University of South Carolina**

**South Carolina Center for Excellence in the  
Assessment of Student Learning**

**ASSESSING STUDENT LEARNING IN MATHEMATICS**

**Foreword**

This report is part of a series of documents prepared by the South Carolina Center for Excellence in the Assessment of Student Learning (CEASL) to describe assessment of student learning in various school curriculum areas from prekindergarten through grade twelve. The focus of this document is assessment of student learning in the mathematics classroom. The report begins with an overview of current curriculum goals in mathematics education and rationale for reforming the content, instructional practices, and assessment methods in school mathematics. The subsequent sections describe alternative methods of assessing mathematics learning that are consistent with the new curriculum and instructional goals set forth for mathematics education. A description of various methods for scoring and grading student work follows the information on assessment methods. Lastly, a brief summary of activities in performance assessment from selected states is provided.

The South Carolina Center for Excellence in the Assessment of Student Learning was established by the South Carolina Commission on Higher Education and is supported by the South Carolina Commission on Higher Education and the College of Education, University of South Carolina. The purpose of this Center is to increase awareness among teacher-educators of recent efforts to change approaches used to assess students' learning in pre-school through high school, and to encourage and support efforts to enhance training in testing, measurement and the assessment of students' learning for preservice educators. The Center is based on the educational philosophy that the fair, accurate and informative assessment of students' learning is an integral part of the teaching-learning process.

Comments or suggestions concerning the information of this report are welcome and may be directed to the authors at the Center.

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## Overview of Current Curriculum Goals

Calls for reforming mathematics curriculum, instruction and assessment have been loud and increasing over the last ten years. Studies and reports such as An Agenda for Action (National Council of Teachers of Mathematics [NCTM], 1980), A Nation at Risk (National Commission on Excellence in Education, 1983), Everybody Counts (National Research Council [NCR], 1989), and the National Assessment of Educational Progress Trial State Assessment Program (National Center for Education Statistics, 1991) have called for major improvement in mathematics education. Most recently, the National Council of Teachers of Mathematics published the document, The Curriculum and Evaluation Standards for School Mathematics (1989) setting forth standards and direction that mathematics reform should take for the nineties and beyond. The foundation of this document is the belief that "all students need to learn more, and often different, mathematics and that instruction in mathematics must be significantly revised" (p. 1). A major theme of this reform effort is captured by Resnick and Resnick (1991) who state, "The traditional view that the basics can be taught as routine skills, with thinking and reasoning to follow later, can no longer guide our educational practice" (p. 39).

According to the NCTM (1989) and the National Research Council (1989), there are several reasons driving the call for reform in mathematics education. Two of the major reasons include the transformation of society from industrial to informational and new developments in the field of cognition on how children learn and process information. Each of these reasons is explicated in more detail below.

Substantial changes have occurred in society since World War II creating new and different mathematics expectations for all members of society. Rapid advancements in technology are transforming our nation from a predominantly industrial society to an informational one. No longer are skills in basic arithmetic computation sufficient for success in the workplace. Today, students must learn to think and reason logically, apply a variety of techniques to approach and solve problems, explore complex open-ended problems, communicate mathematically with others and apply electronic technology to access, use, produce and communicate information. Clearly, the curriculum of school mathematics must change in response to the technological shift and to meet the economic needs of today and tomorrow.

A second major reason for reforming mathematics education is that we have gained new understandings of how children learn and process information. Recent research in learning and cognition reveals that children must be actively engaged in learning experiences in order to process and integrate new concepts and skills with prior information (Baroody & Ginsburg, 1990; Carpenter, Moser, & Romberg, 1982; Ginsburg, 1983). More specifically in mathematics, Lindquist (1989) stresses that:

Learning mathematics is a constructive rather than a passive activity. When students are using prior knowledge to construct new mathematical knowledge, they are learning mathematics. Otherwise, they are receiving a body of knowledge, and often in unrelated and unorganized pieces, which makes it difficult to retrieve and use (p.3).

Resnick and Resnick (1991) support the constructivist position when they argue that educational practice cannot rely on stimulus-response psychology and behavioral objectives to bring about meaningful mathematics learning.

Experts in mathematics education argue that instruction in mathematics must change to reflect the new understandings of how students learn mathematics (NCTM, 1989; Resnick & Resnick, 1991; Romberg & Carpenter, 1986). Mathematics instruction must emphasize problem solving and the interrelatedness of mathematical ideas. Teachers must recognize the value of informal knowledge structures that children bring with them into the classroom. Mathematics instruction can no longer focus on rote memorization of number facts and computation. The goal must be the development of understanding of mathematics concepts and insight into how and why the processes students learn to use will work in both classrooms and applied settings. A wide variety of mathematics materials, resources, and experiences, including technology and concrete models and manipulatives, must be available to enable all students to gain mathematical power and become mathematically literate (NCTM, 1989).

New curriculum and instructional goals in mathematics (NCTM, 1989) have been outlined that stress the importance of such outcomes as problem solving ability, understanding of concepts, and the ability to communicate mathematically and work with others. Baron (1991) contends that in order to achieve these goals, student assessment must reflect the value of student learning in these areas. The traditional methods that have been used to assess mathematics competence such as single answer, multiple choice tests are inadequate. These traditional assessments cannot evaluate a student's ability to reason, to try alternative strategies, to explore and model mathematical ideas with physical objects, or to work cooperatively with others. The following section describes alternative approaches in assessing mathematics learning that are closely aligned with the new curriculum and instructional goals for mathematics education.

### Alternative Methods of Assessment

The search for new approaches and methods of assessing mathematics that are consistent and aligned with newly emerging curriculum goals finds models and strategies in different fields and disciplines. Assessment methods commonly used in music and art are beginning to be used in mathematics to assess student learning. These methods include performance based assessment which requires students to demonstrate what they know and are able to do, and the use of "portfolios" of student work. These assessment approaches seem to provide a more comprehensive understanding of students' capabilities. In addition, they have the potential to more accurately reflect rather than distort curriculum and instruction

in contrast to traditional testing strategies which frequently reshape curriculum and instruction. Performance based assessment and portfolio assessment as they relate to mathematics education are defined and described in subsequent sections of this report.

### ***Performance-based Assessment***

Performance-based assessment in mathematics involves students actively "doing" mathematics. Students may work individually or in groups on mathematical tasks which go beyond traditional exercises in textbooks in some way. The tasks may deal with school mathematics topics in an applied context or they might provide opportunities to conduct mathematical investigations as would a mathematician. The teacher can then assess what the student actually knows and can do by observing an actual performance or behavior, by collecting additional information through questioning the student (or group), and by reviewing the products of student work. Baron (1991) and Stenmark (1991) cite several characteristics of performance tasks in mathematics. These are shown below.

#### **Characteristics of Mathematics Performance Tasks**

- 1) Tasks permit multiple solutions and/or multiple approaches for solving the problem.
- 2) Tasks are engaging and applicable to the real world.
- 3) Tasks allow teachers to examine the process used and the final product.

The following can be used as methods of performance assessment in mathematics: open-ended questions, investigation tasks, project assignments, writing activities, teacher observations and interviews, and enhanced multiple choice questions. Each performance assessment method will be described in what follows and an example will provide illustration.

### ***Open-ended Questions***

Open-response questions and open-ended questions are two types of performance assessment in mathematics which require students to construct and communicate information using their own words, symbols, drawings, figures, and tables. Open-response questions usually pose a problem having one particular answer but students must show work or describe solutions rather than merely choosing the correct answer from a list of alternatives. In contrast, open-ended questions pose problems that have a variety of appropriate solutions which can be found using a variety of approaches. The California Assessment Program describes open-ended questions as having the following four characteristics.

### Characteristics of Open-Ended Questions

- 1) Present an interesting and engaging situation involving several mathematics concepts.
- 2) Allow multiple entry points to accommodate students at various levels of ability and experience.
- 3) Allow multiple solutions to encourage creative responses and diverse paths leading to a solution.
- 4) Provide an audience in which to create a need for students to communicate effectively. (California Department of Education, 1991).

The following sample question from the California Assessment Program is an open-ended question presenting a situation in which a variety of strategies may be used to arrive at a solution. The problem statement also identifies a specific audience to whom the student must respond.

#### Example.

A friend of yours just moved to the U.S. and must ride the bus to and from school each day. The problem is that your friend does not know how to count American money.

Help your friend find the right coins to give the bus driver. A bus ride costs 50 cents. Exact change is needed, and only nickels, dimes, and quarters may be used.

Draw and write something on the sheet of paper that will show and explain to your friend what coins may be used for the bus ride.

(California Curriculum News Report, 1991).

Open-ended questions provide a window into a student's thinking. A teacher can make inferences about a student's understanding and misunderstanding of mathematical concepts by examining the responses to such questions reflecting on both the strategies employed to arrive at a response and the response itself.

Teachers may create open-ended questions from traditional "textbook" problems by asking students to provide responses which explain how they found the solution, what it means, why they think their answer is correct, or some other interpretation or reflection about their work. Two booklets, A Sampler of Mathematics Assessment published by the California Department of Education (Pandey, 1991) and Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions (1991) by Jean Stenmark both contain a collection of open-ended problems that can be used at various grade levels as well as procedures for scoring student responses.

## *Mathematical Investigations and Projects*

Mathematical investigations and projects are more involved and complex than traditional multiple choice or open-ended problems. The task may be completed over an extended period of time, or may even be completed outside of the classroom. Assessment of students' mathematical learning, in such cases, can become an on-going activity. Students might collaborate with one or more partners in completing the task, or students might give progress reports to one another about individual efforts and solicit input and suggestions about what is being done. Such on-going assessment may involve the teacher, classmates, or both the teacher and the students' classmates. Extended investigations and projects facilitate continuous assessment. They allow the teacher to observe and determine how well students employ mathematical thinking, how they collect and organize information and utilize various tools and techniques to communicate mathematically.

The most typical method of assessing student learning during a mathematics investigation is to systematically collect and evaluate samples of student products such as student writings, tables, graphs, diagrams, and responses to extended questions. A teacher may also record direct observations of students or videotape student performance. Students have opportunities to see the approaches other students use during the investigation when the peer dimension is added to the assessment plan. Working with peers helps develop confidence and affective outcomes such as creativity, leadership and cooperation skills, persistence and open-mindedness are often observed (Pandey, 1991).

### Example.

The Learning Technology Center at Vanderbilt University has developed a videodisc based program entitled "The Adventures of Jasper Woodbury" (1991) which presents interdisciplinary units that provide mathematical investigations in a real world context. For example, in the "Big Splash", students must devise a business plan for securing a school loan to set up a dunking booth at their school fair. Each adventure in the Jasper Woodbury Series can take from one to five days to complete. Mathematics skills checklists in the teacher's guide identify skills students might use during the investigation and support assessment through observation of students during work sessions or through examination of work the students submit. For more information on the Jasper Woodbury Series the reader may contact The Optical Data Corporation.

## *Writing Activities in Math*

Using students' writing for purposes of assessment has recently been identified as a valuable technique in mathematics classrooms (Connolly & Vilardi, 1989). Lester and Kroll (1991) describe three categories of writing used for assessment: self-reports, in-class and homework assignments, and paragraphs or essays related to quizzes or tests. Students' self-reports are appropriate methods for assessing student attitudes and beliefs about math. Writing assignments in class or for homework offer students an opportunity to reflect on their own understanding of new concepts or material and also provide the teacher with instructional feedback. Teachers are discovering that when students write about mathematics they begin

to value what they are learning and attempt to relate new learning with prior knowledge (D.J. Clarke, D.M. Clarke, & Lovitt, 1990).

Example.

Azzolino (1990) describes a variety of ways in which writing tasks may be used in mathematics classrooms. For example, the following question could be used as a prompt for students to assess their own capabilities in problem solving:

Is the type of problem you are working on easy or hard for you?  
What makes the problem easy? Hard?

Teachers can read students' responses and note strengths and weaknesses of various students as well as their levels of self confidence and mathematical reflection in solving mathematics problems.

*Observations/Interviews*

Many experts (Lankford, 1974) have observed that the use of multiple choice, single answer tests in mathematics has lead to the neglect of the idea of "listening" to students as an important aspect of teaching and assessment in mathematics. They contend that teacher observations and interviews are important tools that encourage communication from the student to the teacher. Knowledge of a student's thinking, understandings, learning styles, attitudes, and beliefs about mathematics can be determined through carefully conducted observations and oral interviews (Lankford, 1974; Randall, 1987). The literature suggests that developing systematic methods of recording observations and interviews is essential in documenting student learning (Lester & Kroll, 1991). Records may be kept by keeping a journal of observations, writing comments on "stick on" notes that can be transferred to student folders, and using a checklist or other form where items can be checked off or rated (Stenmark, 1991).

Examples.

The following two examples show a checklist and a rating scale that may be used to document observations in problem solving.

Problem-solving Observation Checklist	
Student _____	Date _____
___	1. Likes to solve problems
___	2. Works cooperatively with others in the group
___	3. Contributes ideas to group problem solving
___	4. Perseveres-sticks with a problem
___	5. Tries to understand what a problem is about
___	6. Can deal with data in solving problems
___	7. Thinks about which strategies might help
___	8. Is flexible-tries different strategies if needed
___	9. Checks solutions
___	10. Can describe or analyze a solution

Problem-solving Observation Rating Scale			
Student _____	Date _____		
	Frequently	Sometimes	Never
1. Selects appropriate solution strategies	_____	_____	_____
2. Accurately implements solution strategies	_____	_____	_____
3. Tries a different solution strategy when stuck	_____	_____	_____
4. Approaches problems in a systematic manner (clarifies the question, identifies needed data, plans, solves, and checks)	_____	_____	_____
5. Shows a willingness to try problems	_____	_____	_____
6. Demonstrates self-confidence	_____	_____	_____
7. Perseveres in problem-solving attempts	_____	_____	_____

Source: Randall, (1987), p.18

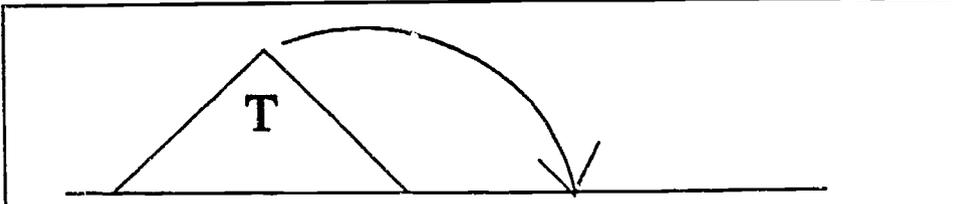
Charles Randall in How to Evaluate Progress in Problem Solving (1987) and Jean Stenmark in Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions (1991) illustrate examples and provide further helpful hints in conducting and recording teacher observations and interviews in the classroom.

### *Enhanced Multiple Choice Questions*

The enhanced multiple choice format presents an engaging stimulus such as a geometric diagram, a series of graphs, or a narrative of a mathematical phenomena. The stimulus is followed by a series of multiple choice questions in which the students must analyze information, integrate concepts, and apply various mathematical skills in order to select the appropriate answer for each question. Although enhanced multiple choice questions are not generally considered a type of performance assessment, they can assess student learning at a higher level than merely recalling facts or procedures. Enhanced multiple choice questions are more complex than traditional multiple choice questions because they require students to understand and apply several mathematical concepts, make connections between problems, and use various mathematics strategies to solve the problem.

#### Example.

In the following example of an enhanced multiple choice question from the California Assessment Program (1989), the student must use spatial reasoning, multiple pattern recognition, and shape representation in order to arrive at the correct answer (Pandey, 1991). Because enhanced multiple choice questions can be easily and objectively scored, many states are developing these questions for use on their statewide mathematics tests. For more information the reader is directed to a later section entitled, "State Activity."



A piece of cardboard shaped like an equilateral triangle, with each side 6 cm in length, is rolled to the right a number of times. If the triangle stops so that the letter "T" is again in the upright position, which of the following distances could it have rolled?

a. 24 cm   b. 30 cm   c. 60 cm   d. 90 cm

Source: Pandey, (1991), p. 17

### *Portfolios*

The use of portfolios is a relatively new approach in assessing student learning in mathematics. A mathematics portfolio is a collection of student work usually completed over a period of time that serves as documentation and /or evaluation of the student's progress. Portfolios are proving to be a valuable tool not only in assessing student's mathematical thinking and understanding, but also in assessing students' reasoning ability and mathematics dispositions such as curiosity, persistence, risk-taking and self confidence (NCTM, 1989; Pandey, 1991; Stenmark, 1991).

Student portfolios provide many advantages over traditional pencil and paper testing of student knowledge. For example, portfolios provide evidence of knowledge and understandings far beyond facts and recall of procedures, they provide a permanent record of a student's progress, they give a clear and overall picture of a student's ability, they allow for different styles of learning, and they provide opportunities for improving the self image and confidence of all students by engaging students in assessing and selecting their own work (Stenmark, 1991).

There are several issues that must be addressed when setting up a portfolio system (Arter & Spandel, 1992; Stenmark, 1991). For example, the purpose, type, contents, and method of evaluating the portfolio are important factors to consider when planning to implement portfolios in the mathematics classroom.

The intended purpose of the portfolio must be decided and clearly defined. That is, are the portfolios being used for evaluation of individual students and classroom instruction, or for the evaluation of curriculum programs district-wide. The purpose of the portfolio influences the type of portfolio that should be used and the contents that should be included.

Experts discuss several different types of portfolios for use in mathematics classrooms. For example, a "working" portfolio is a collection of all the student's work during a specific unit or over a grading period. A student's produced portfolio entries might include responses

to open-ended questions and investigations, reports, charts and diagrams of problem solving processes, mathematics journal entries, group reports, computer generated examples of work, and awards and prizes for mathematics achievement. At a given time, students may review their work portfolio individually, in groups, or with the teacher and select certain pieces to create an "assessment portfolio".

The contents and categories of entries in a portfolio vary widely. Stenmark (1991) states that the focus of the contents in a student assessment portfolio should reflect student thinking and reasoning, growth over time, mathematical connections, student views of themselves as mathematicians, and evidence of problem solving processes. Pandey (1991) suggests that assessment portfolios should contain five to ten pieces including several pieces of student's individual work, a group investigation report, an individual reflective or imaginative piece and one or two pieces selected by the teacher. The statewide portfolio evaluation program in Vermont (Looking Beyond the Answer, 1990-1991) describes four categories for portfolio entries selected by students and teachers. These categories are situational applications, investigations, nonroutine problems, and journals, essays and other entries. The California Assessment Program (California Department of Education, 1990) plans to use the "rule of thirds" in guiding the use of statewide assessment portfolios, that is, one third of the contents will be determined by the student, one-third by the teacher, and one-third by the California Assessment Program Committee.

Methods of evaluating portfolios range from teacher comments or narration on samples of student work to assigning "grades" based on a holistic or analytic scoring approach. Descriptions of methods of evaluating student work appropriate for portfolio assessment and the assessment of performance tasks in mathematics are discussed below.

### Evaluating Student Work

Teachers have traditionally focused on only the final answer when evaluating student work in mathematics. This practice was encouraged by the widespread use of the multiple choice tests in which the responses are scored either right or wrong. More subjective judgements for evaluation are required for the types of assessment being called for in mathematics today, such as open-ended questions, performance tasks, and portfolios. Much more emphasis is placed on the process used to solve problems and students' ability to communicate their thinking and reasoning than on the single final answer.

Two general procedures for scoring students' responses on mathematics tasks are described by Randall (1987) and others who have worked on use of the alternative assessment approaches in mathematics. These are analytic scoring and holistic scoring. Both procedures utilize a scoring rubric or rule which may be a rating scale, checklist or some written standard of performance against which the student work is judged (Randall, 1987). Analytic scoring categorizes the performance on the task into separate dimensions and point values are assigned to each dimension based on the specified criteria. The point values may simply be 1= "inadequate", 2 = "satisfactory", 3= "exemplary" or more elaborate categories might be defined for each score.

### Examples.

The following example illustrates a general analytic scoring scale that can be applied to various problem solving tasks.

<b>Analytic Scoring Scale</b>	
<b>Understanding</b>	0: Complete misunderstanding of the problem 1: Part of the problem misunderstood or misinterpreted 2: Complete understanding of the problem
<b>Planning a Solution</b>	0: No attempt, or totally inappropriate plan 1: Partially correct plan based on part of the problem being interpreted correctly 2: Plan could have led to a correct solution if implemented properly
<b>Getting an Answer</b>	0: No answer, or wrong answer based on an inappropriate plan 1: Copying error; computational error; partial answer for a problem with multiple answers 2: Correct answer and correct label for the answer

Source: Randall, (1987), p. 30

Pure holistic scoring or general impression (Randall, 1987) scoring relies on the evaluator's view of the student's written response when compared to a set of anchor papers. Anchor papers or preselected papers represent the typical response for each score point (e.g. 0,1,2,3,4) and are used to classify students' responses. Focused holistic scoring (Randall, 1987) uses embedded criteria or prescribed characteristics for assigning a total score to the response. An example of a focused holistic scoring scale for mathematics appears in Appendix A.

### State Interest and Activity in Performance Assessment

As of 1991, twenty-one states were implementing, developing, and/or exploring possibilities of alternative assessment procedures (Aschbacher, 1991). This list has undoubtedly grown since 1991 and continues to grow quite rapidly. California, Hawaii, Maine, Massachusetts, New York, North Carolina, Arkansas, Arizona, Connecticut, New Jersey, and Vermont have in place or are piloting performance assessments in mathematics through the use of portfolios and open-ended questions. The following is a brief summary of selected states' activities.

#### **California**

The California Assessment Program (CAP) is revising the state mathematics assessment to reflect and support curriculum reforms. Four types of assessment have been planned and will be implemented into the program depending on the availability of resources. These include

open-ended questions, enhanced multiple choice, investigations, and portfolios. Open-ended questions on the Grade 12 assessment are already in place and are planned for all grade levels. Mathematics investigations and portfolios are planned for full implementation in three to five years.

### *Vermont*

In 1990-1991 Vermont piloted portfolio assessment in the fourth and eighth grades. Students and teachers selected "best pieces" emphasizing reasoning, problem solving, and communication in math. Those involved in this work concluded that the pilot year was useful and instructive and that mathematics portfolios provided a window to view and evaluate mathematics programs. This work suggested the need for additional specifications for the types and amount of content included in the portfolios in order to fairly evaluate student performance (Looking Beyond the Answer, 1990-1991).

### *North Carolina*

The North Carolina Department of Public Instruction is in the process of field testing open-ended test items in reading, social studies, and mathematics in grades three through eight (N.C. Dept. of Public Instruction, 1992). These items will be used as part of their end-of-grade tests that will replace the California Achievement Tests in 1993. Open-ended test questions will focus on higher order thinking skills integrating knowledge and skills from more than one curriculum goal or discipline. The questions will require responses that are truly open (multiple solutions) and students will be required to write out their thought processes and solutions. A sample scoring guide for released open-ended items is available by contacting the N.C. Department of Public Instruction.

### *Massachusetts*

In 1988 the Massachusetts Department of Education included open-ended questions on one form of test in their second biennial assessment (Badger & Thomas, 1989). A sampling of fourth, eighth, and twelfth grade students received a test with some open-ended questions in the areas of reading, math, science, and social studies. The questions in mathematics covered patterns and relationships, geometry, measurement, and numerical and statistical concepts. Results showed that many students in all grade levels were unable to explain basic mathematics concepts or applications. Furthermore, many students did not even attempt to answer the questions. The Massachusetts Department of Education suggests that perhaps these findings reveal that students believe mathematics is more computation and not relevant to their everyday lives.

## ***South Carolina***

The Twelve Schools Project in South Carolina has designed and field tested many open-ended mathematics tasks as well as scoring rubrics. The teachers have received training through in-service and national and state conferences on writing mathematics performance tasks and developing accompanying scoring rubrics. At the current time most of the assessment is being conducted at the classroom level.

In addition, the South Carolina Curriculum Frameworks recommend a criterion-referenced performance assessment to determine how well South Carolina students are performing against agreed upon standards (South Carolina Department of Education, 1992). This assessment would take place at the beginning of the year in grades 4 and 8. The content of the assessment instrument in mathematics would be performance-based and as authentic as possible allowing free access and unrestricted use of mathematics materials such as calculators, measurement devices, geometric models and other manipulatives. The assessment would be scored by a panel of mathematics "readers" using a carefully designed scoring rubric. The Frameworks Committee also recommends that a new exit examination be developed in mathematics that includes open-ended problems and the use of normal tools in mathematics.

South Carolina is also participating in the New Standards Project which is a nationwide effort to create a national examination system. Currently the examination is being designed to consist of three major components: a performance examination, portfolio assessment of student work, and assessment of student projects. The examination will focus on thinking and problem solving skills and the ability to apply knowledge in real life situations.

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## Appendix A

### Focused Holistic Scoring Point Scale

#### 0 Points

These papers have one of the following characteristics:

- \* They are blank.
- \* The data in the problem may be simply recopied, but nothing is done with the data or there is work but no apparent understanding of the problem.
- \* There is an incorrect answer and no other work is shown.

#### 1 Point

These papers have one of the following characteristics:

- \* There is a start toward finding the solution beyond just copying data that reflects some understanding, but the approach used would not have led to a correct solution.
- \* An inappropriate strategy is started but not carried out, and there is no evidence that the student turned to another strategy. It appears that the student tried one approach that did not work and then gave up.
- \* The student tried to reach a subgoal but never did.

#### 2 Points

These papers have one of the following characteristics:

- \* The student used an inappropriate strategy and got an incorrect answer, but the work showed some understanding of the problem.
- \* An appropriate strategy was used, but--
  - a) it was not carried out far enough to reach a solution (e.g., there were only 2 entries in an organized list);
  - b) it was implemented incorrectly and thus led to no answer or an incorrect answer.
- \* The student successfully reached a subgoal, but went no further.
- \* The correct answer is shown, but--
  - a) the work is not understandable;
  - b) no work is shown.

#### 3 Points

These papers have one of the following characteristics:

- \* The student has implemented a solution strategy that could have led to the correct solution, but he or she misunderstood part of the problem or ignored a condition in the problem.
- \* Appropriate solution strategies were properly applied, but--
  - a) the student answered the problem incorrectly for no apparent reason
  - b) the correct numerical part of the answer was given and the answer was not labeled or was labeled incorrectly;
  - c) no answer is given.
- \* The correct answer is given, and there is some evidence that appropriate solution strategies were selected. However, the implementation of the strategies is not completely clear.

#### 4 Points

These papers have one of the following characteristics:

- \* The student made an error in carrying out an appropriate solution strategy. However, this error does not reflect misunderstanding of either the problem or how to implement the strategy, but rather it seems to be a copying or computational error.
- \* Appropriate strategies were selected and implemented. The correct answer was given in terms of the data in the problem.

Source: Randall, (1987), p.34

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